Proceedings of the American Academy of Arts and Sciences.

VOL. L. NO. 7.—JANUARY, 1915.

CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

THE INFLUENCE OF THE MAGNETIC CHARACTERISTICS OF THE IRON CORE OF AN INDUCTION COIL UPON THE MANNER OF ESTABLISHMENT OF A STEADY CURRENT IN THE PRIMARY CIRCUIT.

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By B. Osgood Peirce.†

Presented January 13, 1915. Received Dec. 1, 1914.

Introduction.— Nearly sixty years ago, Helmholtz proved experimentally that the predictions of the mathematical theory which had been constructed to explain the courses of currents in inductively connected circuits, were fulfilled when the inductances were fixed constants, and shortly thereafter Du Bois Reymond put the equations for the currents in the two circuits of an induction coil with air core, into the forms in which they appear in modern textbooks. This simple theory, however, was soon found to be inapplicable when iron cores were used, and we now know that we cannot predict the march under given applied electromotive forces, of the currents in a number of coils wound on an iron core, without a knowledge of the magnetic history of the iron as well as of its magnetic properties, and that a general mathematical theory — if it were possible to form one — would be very complex. Notwithstanding this, it is often necessary to study the magnetic characteristics of a piece of iron by making it the core of a simple induction coil or transformer, and then determining experimentally the effect of its presence upon the manner of growth of the current in each circuit when a given electromotive force is applied to the primary. When the mass of the core is large, it is often important that the observer shall have at the outset a general knowledge of the manner in which the currents will change under a given set of initial conditions; this paper discusses briefly a few typical cases.

Magnets Employed in the Experiments.— In some of the experiments recorded in this paper, I have made use of an electromagnet (J, Fig. 1), the core of which, built up of varnished sheets of iron 0.38 millimeters thick, has a cross section of upwards of 150 square centi-

† Deceased.

meters. The building-up curve of a current in the coil of this magnet is at the beginning slightly modified by the presence of eddy currents in the iron, but the effect is small. When this magnet is put a good many times successively through a cycle with a maximum excitation that will cause a flux density of about 14000, the throws of a ballistic galvanometer used to measure the total flux changes may differ from each other as much as 1% for different rounds, but the averages of a set of measurements made in the first case when the current is put on suddenly and in the second case when it is brought gradually in perhaps half a minute to its full value are practically indistinguishable.

Through the kindness of Dr. George Ashley Campbell I have been allowed to use also a number of toroids belonging to the American Telegraph and Telephone Co., the cores of which are made of wire only $\frac{1}{10}$ of a millimeter in diameter. A hysteresis diagram for any of these coils obtained by a step-by-step ballistic method, agrees exactly, within the limits of my measurements, with a similar diagram obtained by computation from an oscillograph record of a current curve in the coil. As we shall see this fact makes it possible, when

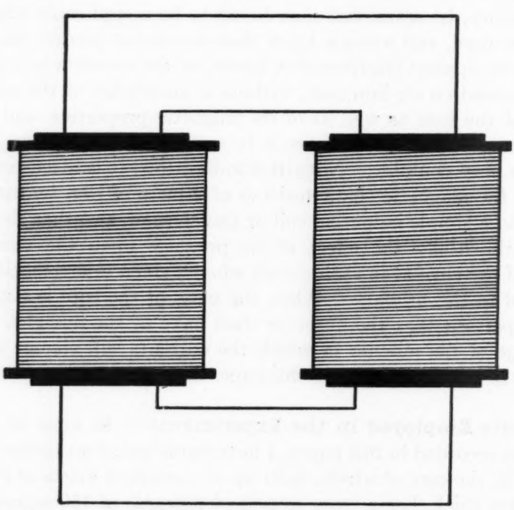

FIGURE 1. The electromagnet J, which has a laminated core of square cross-section of about 156 square centimeters area, and is built up of soft iron plates about one third of a millimeter thick.

one has a hysteresis diagram for a given magnetic journey of the core, to predict accurately the form of a current curve corresponding to this journey under given electric conditions.

In the case of a commercial transformer, eddy currents in the laminated core affect slightly the form of a current curve at the very beginning, but nothing of the sort occurs in coils such as are used for loading telephone circuits. I shall assume, therefore, that the form of a current curve can be foretold if we have a permeability of the core for the given magnetic path, in the cases of such induction coils as we consider here.

The theory and method of investigation here proposed applies accurately to telephone loading coils with finely divided cores, and gives good approximation to correct results with commercial closed-core transformers.

The investigation comprises two parts: Part One, the establishment of the current in the primary coil of a transformer when the secondary is open; and Part Two, the growth of primary and secondary current, when the secondary is closed.

PART I. SECONDARY OPEN.

Theory.— Given a closed-core electromagnet of any form, perhaps one like J shown in Figure 1, suppose that the manner of growth of the flux of induction N in the core as the strength of the exciting current is made to grow larger by steps has been determined when the condition of the core at the outset is a certain definite one. Let the flux N be plotted against the ampere turns T of the exciting current, Curve $O\ B\ A$, Figure 2, and let one horizontal unit of the diagram represent a ampere turns, and one vertical unit represent m Maxwells. If the slope of the curve $O\ B\ A$ at any value T of ampere turns be $\Psi\ (T)$, then

$$\Psi\ (T) = \frac{m}{a}\frac{dN}{dT} \tag{1}$$

If now an e. m. f. E be applied at the time $t = 0$ to the exciting circuit of resistance r, and if the exciting coil has n turns, and if the strength of the exciting current be i, and if we neglect the flux in the air through the turns of the coil, then

$$E - n\frac{dN}{dt} = r\,i. \tag{2}$$

Let the slope $\Psi\ (T)$ be determined graphically from the curve $O\ B\ A$ described above, as a function of T. In practical units

$$E - \frac{n\,dN}{10^8 dt} = r\,i, \tag{3}$$

or

$$E - \frac{n\,a\,\Psi(T)}{10^8 m}\,\frac{dT}{dt} = \frac{r\,T}{n},$$

whence

$$\frac{n\,a\,\Psi(T)\,dT}{10^8 m\left(E - \dfrac{rT}{n}\right)} = dt \tag{4}$$

Call the left hand side of equation (4) $\Omega\ (T)\ dT$, then

$$\Omega\ (T)\,dT = dt.$$

If now $\Omega\ (T)$, computed from the known quantities of equation (4), be plotted as a function of T with one horizontal unit representing a ampere turns and one vertical unit v units of Ω, the area under the curve, thus plotted, from T_1 to T_2 when multiplied by av is equal to the time in seconds which elapses while the excitation is increasing from T_1 to T_2. It is evident that by taking successive values of T as T_1, T_2, T_3 etc., respectively equal to ni_1, ni_2, ni_3, etc., it is easy to obtain a graphical representation of T (and therefore of i) in terms of t. This gives in the absence of eddy currents the current curve.

We may illustrate the working of the foregoing theory by applying it to the electromagnet J, the form of which is shown in Figure 1.

Application to Magnet J.—With the core of J is magnetically neutral at the outset, the application of a series of currents, each a little stronger than the last, gave the numbers of Table I. With these results carefully plotted the slopes of the resulting curve furnished the numbers of Table II.

TABLE I.

Ampere Turns (T)	Flux of Magnetic Induction N through the Core in Thousands of Maxwells
100	35
200	146
300	386
400	622
500	787
600	929
700	1013
800	1086
900	1137
1000	1176
1100	1208
1200	1238
1300	1262
1400	1285
1500	1309
1600	1331
1700	1352
1800	1371
1900	1390
2000	1409

Figure 2 represents Table I graphically in the full curve, $O\,B\,A$; one vertical unit corresponding to a hundred thousand maxwells, and one horizontal unit to a hundred ampere-turns. There is evidence of slight eddy-currents at the outset, but the effect quickly disappears and the magnetization curve follows closely the march of the current. The slopes of the curve $O\,B\,A$, were found in the following way: After the curve had been drawn on a large scale to represent the numbers of Table I, a zinc templet was made from it, as accurately as possible; this was fastened down on a drawing-board over a large sheet of coordinate paper and the value of the slope $\Psi\,(T)$ was determined by measuring on the paper the position of a straight-edge which was held tangent to the templet at any desired point. These slopes

are recorded in Table II and plotted as the dotted curve of the same figure, Figure 2, with one division of ordinate equal to 200 $\frac{\text{Maxwell}}{\text{Ampere-turn}}$.

TABLE II.

Ampere Turns (T)	dN/dT
0	151
50	308
100	545
150	1255
200	2102
250	2639
300	2596
350	2374
400	2080
450	1786
500	1521
600	1119
700	818
800	617
900	437
1000	344
1100	301
1200	273
1300	251
1400	237
1500	222
1600	215
1800	208
2000	201

If after the core of the electromagnet J has been thoroughly demagnetized, a steady electromotive force E is applied to the exciting coil, which consists of n turns of wire of resistance r ohms, the march of the current can be predicted by the aid of equation (4). The values of the slope $\Psi\,(T)$ substituted in equation (4) permits the computation of $\Omega\,(T)$. In a particular case with $E = 10$, $r = 1$ and $n = 100$, the values obtained for $\Omega\,(T)$ are recorded in Table III, and plotted in the boundary of the shaded area of Figure 3. The actual curve which

bounds the shaded area was plotted on a large scale, one vertical unit corresponding to $10^4\ \Omega\ (T)$ and one horizontal unit to a hundred T, and the area X from $T = 0$, to $T = T$, for a number of different values of T was measured with the aid of a good planimeter in terms of the unit square of the figure. A few values of the areas X are shown in Table III. These areas divided by a hundred express the time in seconds to which T corresponds. The resultant curve, $O\ P\ Q$, Figure 3, is the desired curve for growth of the current.

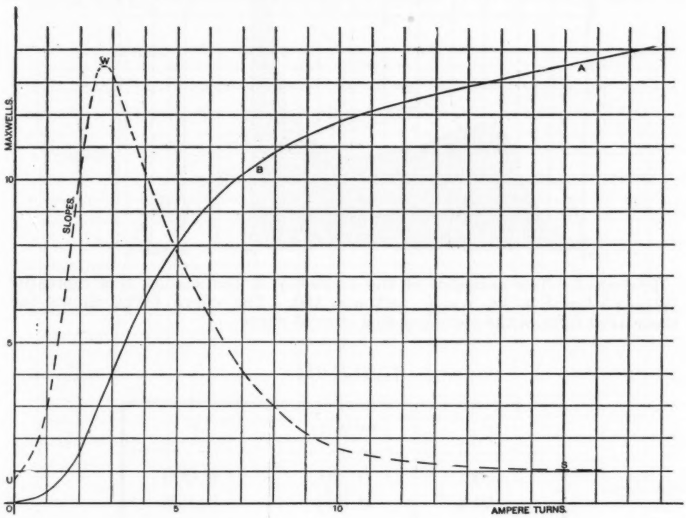

FIGURE 2. Magnetization curve, OBA, for the electromagnet J which at the outset is in a neutral state. Each horizontal division is 100 Ampere-turns and each vertical division is 100,000 Maxwells. The ordinates of the dotted curve represent the slopes of the magnetization curve, with each vertical division equal to 200 $\frac{\text{Maxwell}}{\text{Ampere-turn}}$.

Further Facts Regarding Electromagnet J.— We may add to the foregoing discussion, of the characteristics of the electromagnet J the results of a series of hysteresis cycles obtained some years ago. An outline of this magnet is shown in Figure 1. It weighs about 300 kilograms: the core has a square cross-section of 156 square centimeters area, and was built from varnished sheets of soft iron

about one third of a millimeter thick. The exciting coil consists of 1394 turns of well insulated copper wire.

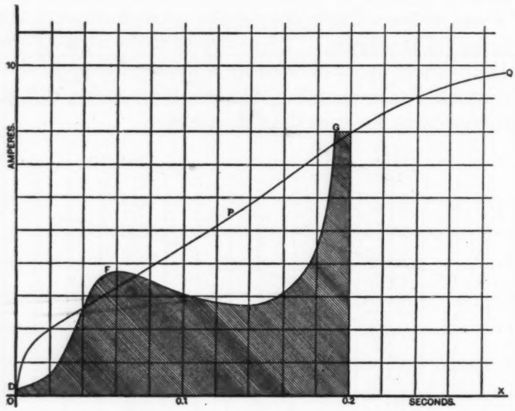

FIGURE 3. The ordinates of the boundary of the shaded area represent $10^4 \Omega(T)$ for $E = 10$, $r = 1$, and $n = 100$. The curve OPQ, shows the theoretical form of the corresponding current curve.

TABLE III.

T	$10^4 \Omega(T)$	X
0	0.151	0.000
50	0.324	0.123
100	0.605	0.336
150	1.477	0.790
200	2.629	1.887
250	3.519	3.477
300	3.710	4.612
400	3.467	8.157
500	3.042	11.404
600	2.800	14.318
700	2.727	17.058
800	3.085	19.884
900	4.370	23.489
950	7.800	26.279

It is difficult to obtain an accurate hysteresis diagram for transformers containing massive iron cores, by the "step-by-step" ballistic method, with a short period galvanometer. Although, eddy currents may be nonexistent the time-lag of magnetization necessitates the use of a long period instrument, when ballistic methods are employed. The galvanometer used throughout these experiments was of the d'Arsonval type, and had a period of about four minutes. The accuracy of the ballistic method was tested by comparing the results, thus obtained, of a corresponding hysteresis cycle for an excitation of

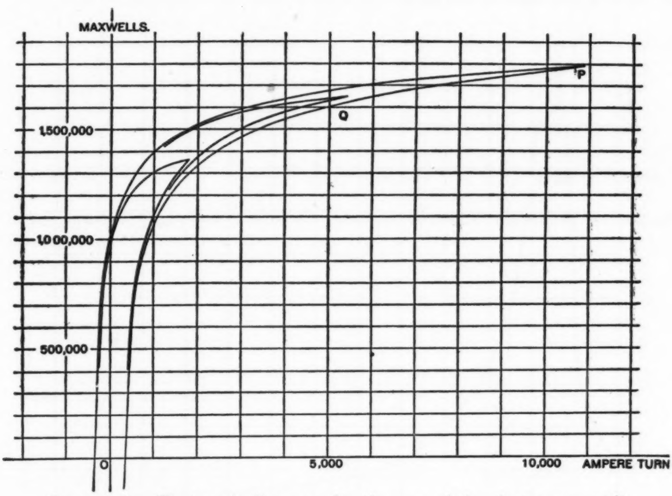

FIGURE 4. Hysteresis diagrams for the core of the electromagnet J.

1812 ampere turns, with a series of results reduced from oscillograph records. Throughout the comparison the agreement lay within a tenth of one per cent, and this may be considered close since it is not always possible to make a large mass of iron travel exactly the same magnetic journey twice.

Computation of Current Growth on Reversal of E. M. F.— Figure 4 shows a series of hysteresis diagrams for the electromagnet J obtained by decreasing and reversing the exciting current step by step after maximum excitations of 1812, 5370, and 10880 ampere-turns. The results of measurements of the flux changes in the core for the

first of these cycles are given in Table IV. The next diagram, Figure 5, shows the slopes of the curve corresponding to Table IV, as a function of the strength of the exciting current.

From equation (4) it will be seen that the march of current on reversal of e. m. f. may be obtained. If the slope for any point of the flux curve is multiplied by $n^2/10^8\,(E{-}ri)$, the result is the value of dt/di, for the reversed current curve, when the constant voltage E is applied to the exciting coil and reversed, where the coil consists of n turns of wire with resistance r ohms. Figure 5 exhibits dt/di for $E = 19.5$, $r = 15$, and $n = 1394$.

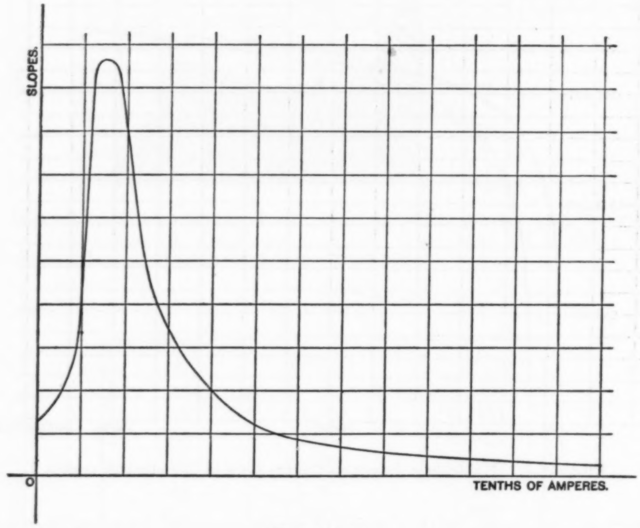

FIGURE 5.

If now the area X, underneath the curve, Figure 7, from $x = 0$ to $x = i$, for a number of different values of i, be measured in terms of the unit square of the figure; this area expresses the time required for the reverse current to attain the strength i. Table V contains a few values of X which were measured with a planimeter, and from which the desired reverse current curve, shown in Figure 7, was plotted.

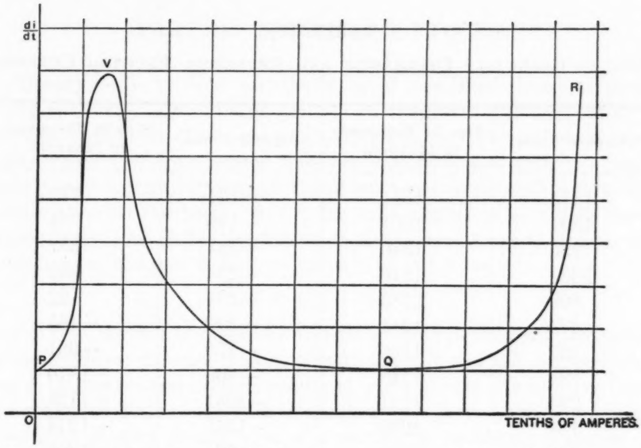

FIGURE 6. The value of di/dt for a reverse current in the coil of the magnet J when $E = 19.5$ and $r = 15$.

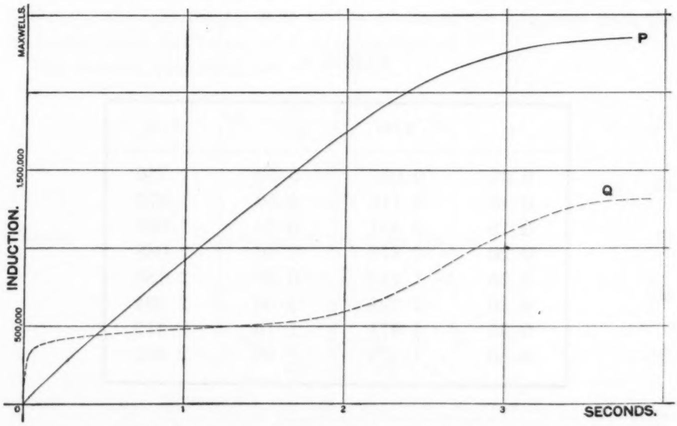

FIGURE 7. The full curve, OP, shows the rate of increase of the flux of magnetic induction through the core of the magnet J while a reverse current of 1.3 amperes is being established in the exciting coil of 1394 turns. The current curve is shown on an arbitrary scale by the dotted line.

TABLE IV.

Flux on Gradually Decreasing and Reversing Exciting Current.

Ampere Turns	Flux in Thousands of Maxwells	Ampere Turns	Flux in Thousands of Maxwells
1812	1371	−131	772
1394	1351	−148	734
1255	1340	−181	552
1031	1316	−234	332
809	1285	−294	22
474	1211	−392	−465
392	1186	−474	−661
294	1148	−809	−1010
234	1121	−1031	−1128
181	1099	−1255	−1214
148	1070	−1394	−1265
131	1060	−1821	−1371
000	953		

TABLE V.

i	$X/10$	i	$X/10$
0.05	0.057	0.50	1.750
0.10	0.115	0.60	1.875
0.15	0.494	0.70	1.985
0.20	0.878	0.80	2.088
0.25	1.141	0.90	2.188
0.30	1.325	1.00	2.294
0.35	1.471	1.10	2.412
0.40	1.579	1.20	2.682

PART II. SECONDARY CLOSED.

Theory.— In an iron core built up of varnished sheets of metal there is usually a noticeable amount of magnetic leakage, but in a toroidal core made of one piece of varnished, very fine, soft iron wire, upon which two transformer coils are uniformly and closely wound, the leakage is generally negligible and we may assume with a close approximation to the truth, that if the coils consist of n_1 and n_2 turns respectively, and if the resistances of their circuits are r_1 and r_2, the excitation of the core in ampere turns is at any time,

$$T = n_1 i_1 + n_2 i_2$$

and if N is the flux of magnetic induction through the core, the corresponding fluxes through the two circuits are $n_1 N$ and $n_2 N$ respectively. Let N be determined experimentally at many points of a journey from one magnetic state of the core to another by a given path, and let the results be plotted in the form of a curve, the unknown equation of which may be written in the form $N = \Phi\,(T)$. Let the slope of this curve be obtained at a large number of points so as to give the values of dN/dT or $\Psi\,(T)$ for many values of T, then it is possible to predict the form of a building up curve for a current in the exciting coil which will carry the core over a part, or the whole, of the magnetic path for which we know the values of N as a function of T.

The current equations are

$$E_1 - \frac{n_1}{10^8} \cdot \frac{dN}{dt} = r_1 \cdot i_1, \tag{5}$$

$$-\frac{n_2}{10^8} \cdot \frac{dN}{dt} = r_2 \cdot i_2, \tag{6}$$

whence

$$E_1 n_2 = n_2 \cdot r_1 \cdot i_1 - n_1 \cdot r_2 \cdot i_2, \tag{7}$$

$$i_1 = \frac{n_1 \cdot r_2 \cdot T + n_2^2 \cdot E_1}{n_1^2 \cdot r_2 + n_2^2 \cdot r_1}, \tag{8}$$

$$i_2 = \frac{n_2 \cdot r_1 \cdot T - n_1 \cdot n_2 \cdot E_1}{n_1^2 \cdot r_2 + n_2^2 \cdot r_1}, \tag{9}$$

$$\frac{n_1}{10^8} \cdot \frac{dN}{dt} \cdot \frac{dT}{dt} = \frac{n_1^2 \cdot r_2 \cdot E_1 - n_1 \cdot r_1 \cdot r_2 \cdot T}{n_1^2 \cdot r_2 + n_2^2 \cdot r_1}, \tag{10}$$

$$\frac{n_1 (n_1^2 \cdot r_2 + n_2^2 \cdot r_1) \cdot \Psi\,(T) \cdot dT}{10^8 (n_1^2 \cdot r_2 \cdot E_1 - n_1 \cdot r_1 \cdot r_2 \cdot T)} \equiv \Omega\,(T) \cdot dT = dt. \tag{11}$$

If now $\Omega (T)$ be accurately plotted against T, the area under this curve from T_1 to T_2 is equal to the time in seconds which elapses while the excitation is growing from T_1 to T_2. If a large number of these areas be measured by aid of a planimeter, it is easy to give a graphical representation of T (and therefore of i_1 and i_2) in terms of t, and we may expect the current curves thus obtained to correspond closely with the oscillograph records for the same case. If r_2, r_2, n_1, n_1, and E_1 are all increased in a given ratio λ, the quantity $\Omega (T)$ and therefore, dt, will be increased in the same ratio.

Application to Transformer with Fine Wire Core.— It will be instructive to apply the foregoing theory to a certain transformer (DN), in which magnetic leakage and eddy currents were negligible. This transformer was constructed in the form of a toroid, about 41 centimeters mean diameter, the core of which was made of about 25 kilograms of fine, soft, varnished iron wire.

After the core of the transformer had been thoroughly demagnetized the magnetic flux through the core, due to an ascending series of steady currents in the exciting coil, was determined. The results of the long series of measurements, which were taken with a slow period ballistic galvanometer, are given in Table VI. The full curve, OK, Figure 8, reproduces the table graphically: one vertical unit corresponds to fifty thousand maxwells, and one horizontal unit to a thousand ampere-turns. The ordinates of the dotted curve exhibit, on an arbitrary scale, the corresponding slopes of the other. A few values of the slope, dN/dT, are given in Table VII.

From the foregoing theoretical discussion, and the numbers given in Tables VI and VII, it is always possible to predict under fixed conditions the growth of the excitation in the core, the march of the current in the primary and secondary coils, and the manner of increase of the flux of magnetic induction in the core of the transformer in question. It will be seen from equation (11), when $E = 10$, $n_2 = 1000$, $n_1 = 100$, $r_2 = 10$, and $r_1 = 1$, that if the slope of any point of the curve, OK, is multiplied by $11/10^4\,(1000-T)$, the result is the value of $\Omega (T)$ for the given value of T. A few values of $\Omega (T)$ are shown in Table VIII. If now $\Omega (T)$ be plotted as a function of T, and the area from $T = 0$ to $T = T_1$, for a number of different values of T be measured in terms of the unit square of the figure, this area gives the time in seconds for the excitation in the core to attain the value T.

The curve, $VWSZ$, bounding the shaded area (Fig. 9), reproduces the first two columns of Table VIII graphically; that is, $\Omega (T)$ as a function of T. For convenience $10^4\,\Omega (T)/5$ was taken as the vertical

unit, and 100 ampere-turns as the horizontal unit. In consideration of the units here chosen the growth of the excitation in the core (in ampere-turns) would be given by the equation

$$t = 20 \int_0^{T_1} \Omega\,(T)\, dT \tag{12}$$

Therefore, it is clear that the area, between the limits of $T = 0$ and $T = T_1$, divided by 20 gives the time in seconds for the excitation to grow from 0 to T_1. The third column of the table contains a number

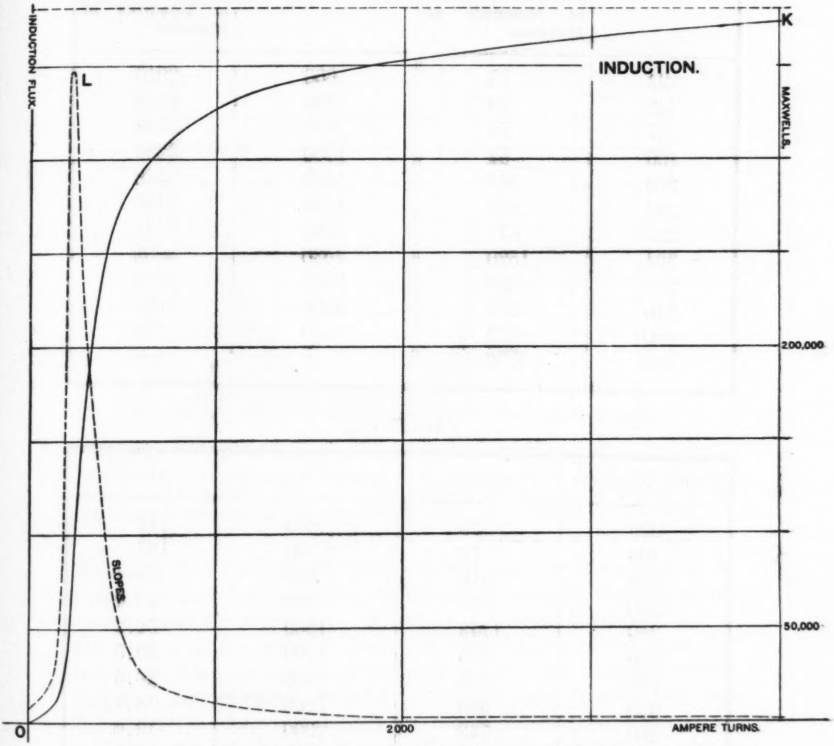

FIGURE 8. Magnetization curve for the finely divided core of the transformer (DN) which at the outset is in a neutral state. The dotted curve represents, on an arbitrary scale the slopes of the other.

of areas, found by mechanical integration, for different values of T and the next column the corresponding time in seconds which the excitation took to rise from 0 to T. The curve $OBLP$ shows graphically the increase of the excitation as a function of the time.

TABLE VI.

Ampere Turns	Flux of Magnetic Induction (N) through the Core in Hundreds of Maxwells	Ampere Turns	Flux of Magnetic Induction (N) through the Core in Hundreds of Maxwells
23	10	600	2910
42	20	700	3032
87	50	800	3124
100	52	1000	3259
200	240	1200	3348
232	445	1400	3406
257	879	2000	3516
294	1520	2630	3602
300	1548	3000	3644
316	1733	3500	3690
400	2372	4160	3740
500	2736		

TABLE VII.

Ampere Turns (T)	dN/dT	Ampere Turns (T)	dN/dT
000	35	600	142
100	76	700	103
200	510	800	79.4
220	700	1000	53.9
240	1742	1200	36.2
260	1670	1400	30.5
280	1338	1600	20.5
320	950	1800	18.7
360	748	2000	13.6
400	552	3000	10.9
500	251	4000	4.8

We may now determine the march of the current in the primary and the secondary coils of the transformer, for the given values of E_1, n_1, n_2, r_1, and r_2. Substituting in equations (21), and (22) we get

$$i_1 = \frac{T + 10^4}{1100}, \qquad i_2 = \frac{T - 10^3}{1100} \tag{13}$$

for any given T. Table VIII, columns five and six, contain values of i_1, and i_2 corresponding to the value of T which is given in the first column. Figure 10: the curve PRH shows graphically the march of

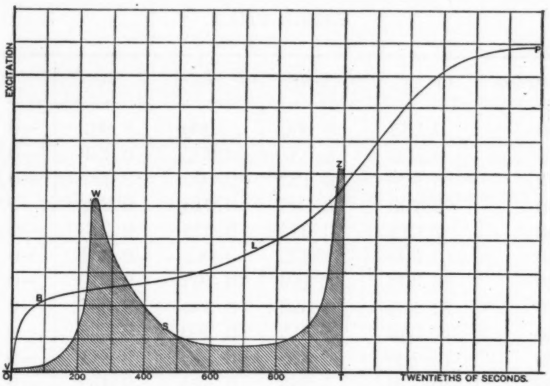

FIGURE 9. The ordinates of boundary of the shaded area represent $10^4 \Omega(T)/5$, $E = 10$, $n_1 = 100$, $n_2 = 1000$, $r_1 = 1$, and $r_2 = 10$, the abscissas correspond to $100T$. The curve OLP shows the manner of growth of the excitation in the core of the transformer (DN), as a function of the time.

the current in the primary coil; and the curve SEC, bounding the shaded area at the bottom of the figure, the manner of decay of the current i_2 in the secondary. It will be seen from the figure that i_1, the current in the primary, builds up very rapidly at the start, but before reaching its maximum value it remains for a comparatively long time almost exactly parallel to the time axis. During this time the indication of an amperemeter in the circuit does not change perceptibly, and yet the flux of magnetic induction through the core is increasing at a very nearly constant rate. The shaded area, bounded by the curve PRH and its asymptote KD, which is a measure of the

induction flux, is constantly growing. If the core of a transformer is very large the building-up time may be a minute or more, and the phenomenon may then become very striking.

The curve ONM (Fig. 10) represents graphically the flux of magnetic induction through the core as a function of the time. One vertical unit of the figure corresponds to thirty thousand maxwells, and one horizontal unit to a twentieth of a second.

A hysteresis diagram for the transformer under investigation is shown in Figure 11. The cycle corresponds to a maximum excitation

TABLE VIII.

Ampere Turns	$10^4\Omega(T)$	Area	t	i_1	i_2
00	0.385	0.000	000	9.092	−0.909
50	0.550	0.037	0.002	9.136	−0.864
100	0.929	0.123	0.006	9.182	−0.818
150	2.700	0.268	0.013	9.230	−0.773
200	7.013	0.692	0.035	9.273	−0.727
230	15.500	1.232	0.062	9.301	−0.700
250	26.300	2.169	0.108	9.320	−0.682
270	22.500	4.093	0.205	9.338	−0.664
300	17.950	5.313	0.266	9.365	−0.636
350	13.100	6.866	0.343	9.411	−0.591
400	10.120	7.988	0.399	9.458	−0.545
500	5.522	9.444	0.472	9.547	−0.454
600	3.905	10.297	0.515	9.636	−0.363
700	3.777	11.037	0.552	9.727	−0.273
800	4.367	11.814	0.596	9.800	−0.182
900	7.095	12.886	0.644	9.910	−0.091
950	12.980	13.786	0.689	9.957	−0.045
980	30.300	14.933	0.747		

of 4200 ampere-turns, and the results of measurements of the flux changes in the core for this cycle are given in Table IX. From what has been explained already, and the data given here one can predict with accuracy the characteristics of the core, and the current curves for this transformer, for any practical case within the limits of the above excitation.

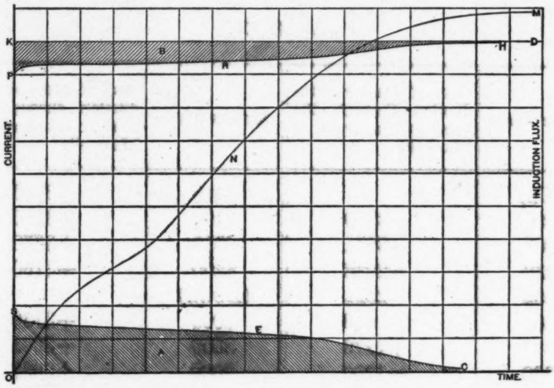

Figure 10. Transformer (DN): The curves, PRH and SEC, deduced from theoretical considerations, indicate the march of the current in the primary and secondary coils respectively. The increase of the flux of magnetic induction, through the core, with the time is shown by the curve ONM.

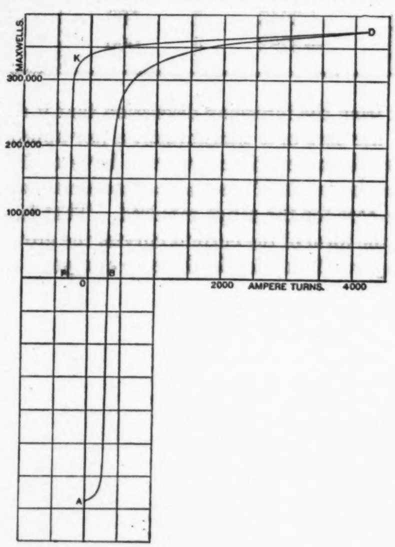

Figure 11. Hysteresis diagram for the core of the transformer (DN).

TABLE IX.

Ampere Turns	Flux of Magnetic Induction through the Core in Hundreds of Maxwells.	Ampere Turns	Flux of Magnetic Induction through the Core in Hundreds of Maxwells
(Up)		1410	+3406
000	−3380	2190	+3580
203	−3241	3310	+3670
238	−2938	4200	+3750
262	−2321	(Down)	
303	+ 866	4000	+3730
317	+1276	3500	+3698
339	−1700	3000	+3667
360	+2030	2500	+3635
386	+2248	2000	+3603
422	+2482	1500	+3571
502	+2752	1000	+3531
685	+3048	500	+3482
1050	+3282	000	+3380

It is evident from the foregoing discussion that if eddy currents are nonexistent in the core of a transformer, a fair approximation to the form which the characteristic curves will have under any given circumstances can be made if one has an accurate statical hysteresis diagram of the core for the range required. That is, one can predict with accuracy the form of the current curves, the growth of excitation, and flux of magnetic induction in the core of a good transformer.

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